DO NOW

Look at and discuss the following chart.

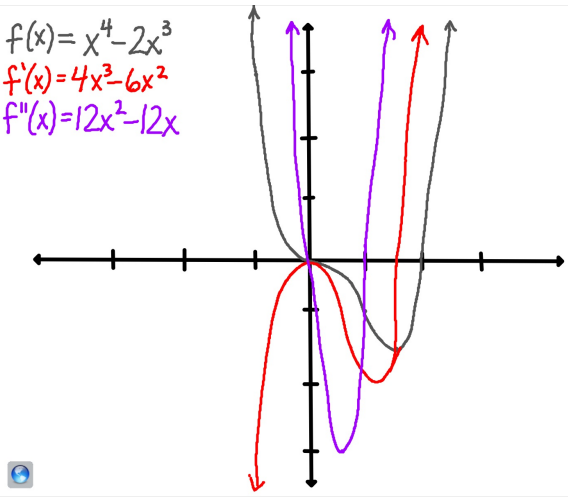
$f(x)$	$f'(x)$	$f''(x)$
increasing	+	
decreasing	-	
relative min. or max.	0	
concave up	increasing	+
concave down	decreasing	-
point of inflection	relative min. or max.	0
	concave up	increasing
	concave down	decreasing
	point of inflection	relative min. or max.
		concave up
		concave down
		point of inflection

Page 1

$f(x) = x^4 - 2x^3$

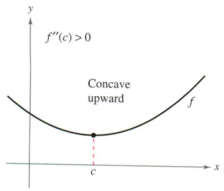
$f'(x) = 4x^3 - 6x^2$

$f''(x) = 12x^2 - 12x$

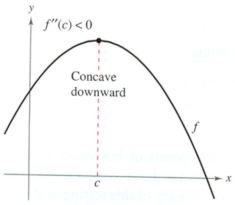


Page 2

4.4 The Second Derivative Test - Day 2



If $f'(c) = 0$ and $f''(c) > 0$, $f(c)$ is a relative minimum.



If $f'(c) = 0$ and $f''(c) < 0$, $f(c)$ is a relative maximum.

Page 3

Theorem: Second Derivative Test

c is a critical number

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then $f(c)$ is a RELATIVE MINIMUM
2. If $f''(c) < 0$, then $f(c)$ is a RELATIVE MAXIMUM

If $f''(c) = 0$, the test fails.

f may have a relative maximum, relative minimum, or neither.

*Use the 1st derivative test

Page 4

$f(x) = x^4$

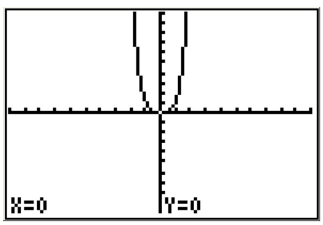
$f'(x) = 4x^3$

Critical #'s → $x = 0$

$f''(x) = 12x^2$

$f''(0) = 0$ ← Test fails

Relative minimum (look at graph)



Page 5

$f(x) = -x^4$

$f'(x) = -4x^3$

$f''(x) = -12x^2$

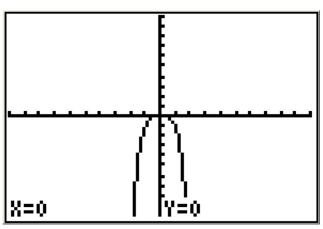
Critical #'s → $x = 0$

$f'(x) = 0$

$-4x^3 = 0$

$f''(0) = 0$ Test fails

Relative maximum



Page 6

$f(x) = x^3$

$f'(x) = 3x^2$

$f''(x) = 6x$

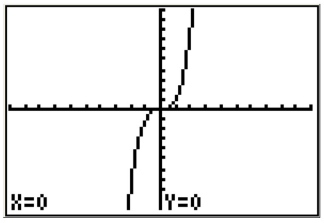
critical #'s

$x = 0$

$f''(0) = 0$

Test fails

Point of inflection



Example: Find all extrema of the function: $f(x) = \frac{1}{4}x^4 - 2x^2$

$f'(x) = x^3 - 4x$

$f''(x) = 3x^2 - 4$

Critical #'s $f'(x) = 0$

$x(x^2 - 4) = 0$

$x(x+2)(x-2) = 0$

$x = 0 \quad x = 2 \quad x = -2$

$f(-2) = \frac{1}{4}(16) - 2(4) = -4$

$f(0) = 0$

$f(2) = -4$

Relative minima: $(-2, -4)$ and $(2, -4)$

Relative maximum: $(0, 0)$

$f''(-2) = +$ ↑ ← relative minimum

$f''(0) = -$ ∩ ← relative maximum

$f''(2) = +$ ↑ ← relative minimum

Example: Find all extrema of the function: $f(x) = x^3 - 9x^2 + 27x$

$f'(x) = 3x^2 - 18x + 27$

$f''(x) = 6x - 18$

No extrema

$f'(x) = 0$

$3x^2 - 18x + 27 = 0$

$3(x^2 - 6x + 9) = 0$

$3(x-3)^2 = 0$

$x = 3$ ← critical #

$f''(3) = 0$ *TEST FAILS

$(-\infty, 3)$	$(3, \infty)$
$f'(0) = +$	$f'(4) = +$

HOMEWORK

pg 235 - 236; 29, 31, 33, 37, 39, 40, 42, 47, 67, 69, 70